J. Fluid Mech. (1980), *vol.* 97, *part* 2, *pp.* 225–237
Printed in Great Britain

Effects of peripheral-layer viscosity on peristaltic transport of a bio-fluid

By J. B. SHUKLA, R. S. PARIHAR, B. R. P. RAO AND S. P. GUPTA

Department of Mathematics,
Indian Institute of Technology,
Kanpur-208016, India

(Received 30 May 1978 and in revised form 19 July 1979)

The effects of peripheral-layer viscosity on the flow characteristics of a bio-fluid due to peristaltic transport has been investigated. It is shown that, for a given pressure drop, the flow flux increases and the frictional force decreases as the viscosity of the peripheral-layer fluid decreases. However, for zero pressure drop, the flux does not depend upon this viscosity while the friction force decreases as it decreases.

The analysis has been applied and compared with observed data.

1. Introduction

The term 'peristalsis' is used for the mechanism by which a fluid can be transported through a distensible tube when contraction or expansion waves propagate progressively along its length. Peristaltic action is used by human organs (such as ureter, male reproductive system, gastro-intestinal tract, bile duct) to perform their function effectively. It has also been suggested that this mechanism might be useful in explaining the function of cilia transport through the *ductus efferentes* of the male reproductive organs, Lardner & Shack (1972) and Sleigh (1974).

Latham (1966) was probably the first to investigate the mechanism of peristalsis in relation to mechanical pumping. Since then, several investigators have contributed to the study of peristaltic action in both mechanical and physiological situations (Burns & Parkes 1967; Hanin 1968; Barton & Raynor 1968; Fung & Yih 1968; Shapiro, Jaffrin & Weinberg 1969; Yin & Fung 1969; Chow 1970; Zien & Ostrach 1970; Li 1970; Lykoudis & Roos 1970; Weinberg, Eckstein & Shapiro 1971; Mittra 1971; Jaffrin & Shapiro 1971; Tong & Vawter 1972. In particular, Burns & Parkes (1967) used perturbation techniques to study the peristaltic motion through a channel and a tube. Barton & Raylor (1968) studied the peristaltic motion in a circular tube by using long and short wavelength approximations. The fluid mechanics of the ureter has been studied by Lykoudis & Roos (1970) and Boyarsky & Labay (1972). The interaction of Poiseuille flow on the peristaltic motion has been studied by Mittra & Prasad (1974). Gupta & Seshadri (1976) have investigated the peristaltic pumping in a non-uniform tube.

In these studies, the effects of viscosity variation of the fluid has not been taken into account, though there have been suggestions that peristaltic mechanism may be involved in vasomotion of small blood vessels, (Fung & Yih 1968), in *ductus efferentes* of the male reproductive tract (Lardner & Shack 1972), in transport of spermatozoa in the cervical canal (Smelser, Shack & Lardner 1974), and in intestines; all being cases where the viscosity of the fluid near the wall is different from its viscosity in the centre of the duct.

Keeping these in view, in this paper investigations are carried out to study the effects of viscosity variation of the fluid on the mechanism of peristaltic transport through a pipe and a channel using the long-wavelength approximation applied by Barton & Raynor (1968).

2. Peristaltic transport through a tube

Let us consider the peristaltic motion of an incompressible Newtonian fluid through a tube whose viscosity varies across the duct, the physical situation of which is shown in figure 1. Since the wall of the tube is executing travelling ring waves due to peristalsis, the geometry of wall sürface can be described as

$$H(X',t') = a+b\sin 2\pi/\lambda(X'-ct'), \tag{1}$$

where a is the mean radius of the tube b is the amplitude and λ is the wavelength of the peristaltic wave, and c is the wave propagation velocity. It may be noted here that the flow is completely symmetrical about the axial co-ordinate, X'.

To study the problem, in what follows we have transformed the stationary co-ordinates, R', X' to moving co-ordinates r', x' (with U', W' and u', w' as respective fluid velocity components in these co-ordinates, see figure 1) which move with the wave velocity, c, in the positive X' direction, as follows:

$$R' = r', \quad X' = x'+ct'; \quad U' = u', \quad W' = w'+c. \tag{2}$$

Using the long wavelength approximation as in Barton & Raynor (1968), Shapiro *et al.* (1969) and Lardner & Shack (1972), and neglecting inertia terms, the equations of momentum and continuity in the moving co-ordinates can be simplified in dimensionless form by the following:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\bar{\mu}\frac{\partial w}{\partial r}\right) = \frac{\partial p}{\partial x}, \tag{3}$$

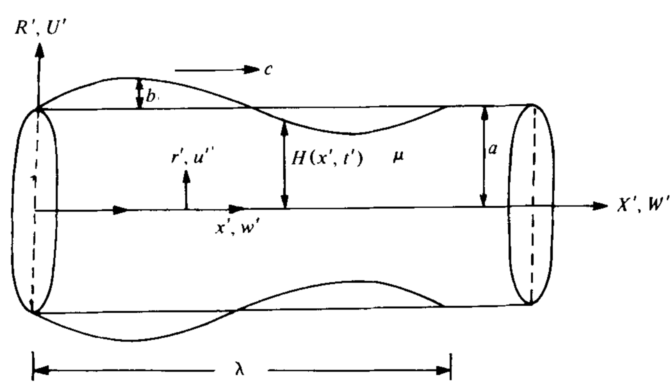

FIGURE 1. Peristaltic transport through a tube.

$$\frac{\partial p}{\partial r} = 0, \tag{4}$$

$$\frac{1}{r}\frac{\partial}{\partial r}(r.u)+\frac{\partial w}{\partial x} = 0. \tag{5}$$

Here

$$\left.\begin{aligned} & r = \frac{r'}{a}, \quad R = \frac{R'}{a}, \quad x = \frac{x'}{\lambda} = X-t, \quad X = \frac{X'}{\lambda}, \\ & t = \frac{ct'}{\lambda}, \quad \bar{\mu} = \frac{\mu}{\mu_1}, \quad p = \frac{p'a^2}{\lambda c \mu_1}, \\ & u = \frac{\lambda}{ac}u', \quad U = \frac{\lambda}{ac}U', \\ & w = \frac{w'}{c} = W-1, \quad W = \frac{W'}{c}, \end{aligned}\right\} \tag{6}$$

where R, X, r, x are dimensionless stationary and moving co-ordinates, t is dimensionless time, $\mu(r)$ is a viscosity function, μ_1 being the viscosity of the central layer and μ_2 that of the peripheral layer, and p is the dimensionless pressure. It is noted from equation (4) that the pressure is approximately a function of x only i.e.

$$p = p(X, t) = p(x).$$

The non-dimensional boundary conditions are:

$$\left.\begin{aligned} \frac{\partial w}{\partial r} &= 0 \\ u &= 0 \end{aligned}\right\} \quad \text{at} \quad r = 0, \tag{7a}$$

$$\left.\begin{aligned} w &= -1 \\ u &= \frac{\partial h}{\partial t} = w|_{r=h}\cdot\frac{\partial h}{\partial x} \end{aligned}\right\} \quad \text{at} \quad r = h = 1+\epsilon\sin 2\pi x \tag{7b}$$

where $h = H/a$, and the dimensionless amplitude of the wave $\epsilon = b/a < 1$.

Integrating equation (3) and using the corresponding boundary conditions (7) for w, we get the dimensionless velocity

$$w = -1+\left(-\frac{1}{2}\frac{dp}{dx}\right)\int_r^h \frac{r}{\bar{\mu}(r)}\,dr. \tag{8}$$

The dimensionless flux $q(= q'/\pi a^2 c$; q' being the flux in the moving system) is given by

$$q = \int_0^h 2\mathrm{r}.w.dr, \tag{9}$$

which, using equation (8), gives

$$q = -h^2-\tfrac{1}{2}\frac{dp}{dx}.f \tag{10}$$

where

$$f = \int_0^h \frac{r^3}{\bar{\mu}(r)}\,dr. \tag{11}$$

Integrating equation of continuity (5) with respect to r and using the boundary conditions (7), it may be noted that q is constant with respect x.

Since the pressure drop, $\Delta p = p(0) - p(\lambda)$, across one wavelength is the same whether measured in the fixed or moving co-ordinate system it can be calculated from equation (10) as follows:

$$\Delta p = -\int_0^1 \frac{dp}{dx}dx = 2q\int_0^1 \frac{dx}{f} + 2\int_0^1 \frac{h^2 dx}{f}. \tag{12}$$

The flux, q, is related with the dimensionless flux, Q, $(= Q'/\pi a^2 c$; Q' is the flux in the stationary co-ordinate system) by the following relation:

$$Q = 2\int_0^h WRdR = 2\int_0^h r(w+1)\,dr = q + h^2. \tag{13}$$

The time averaged flux, $\bar{Q}$, for a complete time period, $T = \lambda/c$, is obtained by using equation (13) as

$$\bar{Q} = \frac{1}{T}\int_0^T Qdt' = \int_0^1 Qdt$$
$$= q + 1 + \tfrac{1}{2}\epsilon^2, \tag{14}$$

where q is given by equation (12).

The dimensionless friction force F, $(= F'/\pi\lambda c\mu_1$; F' is the friction force at the wall in the stationary co-ordinate system which is same as in the moving system), across one wavelength can be obtained by using equation (8), as

$$F = -\int_0^1 h^2 \frac{dx}{dp} dx, \tag{15}$$

which, on using equation (10), gives

$$F = 2\int_0^1 \frac{h^4}{f}dx + 2q\int_0^1 \frac{h^2 dx}{f}. \tag{16}$$

For any given viscosity function $\bar{\mu}(r)$, the flux and the friction force in the tube under peristaltic transport can be investigated from equation (14) and (16) respectively.

3. Effects of peripheral-layer viscosity

To study the effects of peripheral layer on the flow characteristics, consider the viscosity variation in the dimensionless form as (see figure 2):

$$\bar{\mu} = 1 \quad \text{for} \quad 0 \leqslant r \leqslant h_1,$$
$$\bar{\mu} = \bar{\mu}_2 = \mu_2/\mu_1 \quad \text{for} \quad h_1 \leqslant r \leqslant h, \tag{17}$$

where $\mu_2 \leqslant \mu_1$. It is noted that when the geometry of the wall changes to

$$h = 1 + \epsilon \sin 2\pi x$$

in the moving co-ordinate system owing to peristalsis, the corresponding change that would occur in the geometry of the interface which may be given by (see figure 2)

$$h_1 = H_1/a = \alpha + \epsilon_1 \sin 2\pi x, \tag{18}$$

where $\alpha = a_1/a$ is the dimensionless mean radius of the central layer and $\epsilon_1 = b_1/a$, is the amplitude of the interface wave, to be determined.

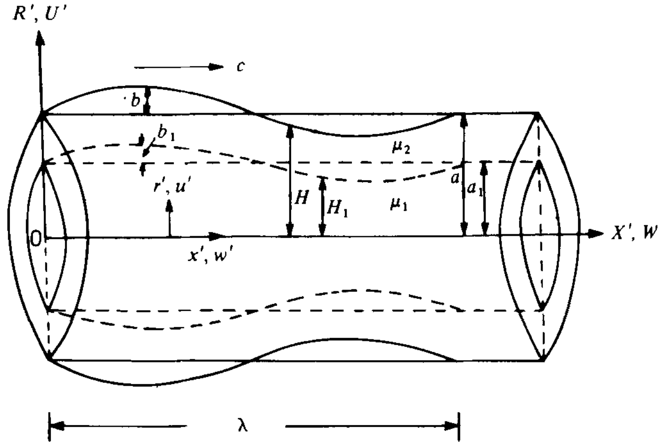

FIGURE 2. Peristaltic motion in a tube with peripheral layer adjacent to the wall.

To determine ϵ_1, we proceed as follows. In the two regions $0 \leqslant r \leqslant h_1$ and $h_1 \leqslant r \leqslant h$, the corresponding non-dimensional fluxes can be obtained, by using (8) and (17) appropriately, as

$$q_1 = \int_0^{h_1} 2rw\,dr$$

$$= -h_1^2 - \frac{h_1^2}{4\bar{\mu}_2}\frac{dp}{dx}\left\{h^2 - \left(1-\frac{\bar{\mu}_2}{2}\right)h_1^2\right\}, \tag{19}$$

$$q_2 = \int_{h_1}^{h} 2rw\,dr$$

$$= -(h^2 - h_1^2) - \frac{1}{8\bar{\mu}_2}\frac{dp}{dx}(h^2-h_1^2)^2, \tag{20}$$

where q_1 is the dimensionless flux of the central layer fluid and q_2 that of the peripheral layer, in the moving co-ordinate system. From (19) and (20) the total flux can be obtained as

$$q = q_1 + q_2 = -h^2 - \frac{1}{8\bar{\mu}_2}\frac{dp}{dx}\{h^4 - (1-\bar{\mu}_2)h_1^4\}. \tag{21}$$

Expression (21) can also be obtained from equation (10) after using the viscosity function given in equation (17). Now, integrating the equation of continuity separately in the two regions, it may be noted that not only q but also q_1 and q_2 are constants with respect to x.

As in the general case, integrating equations (19), (20) and (21) across one wavelength and noting that the pressure drop is same in each case, we have

$$q = q_1 + q_2 = -I_2/I_1 + \Delta p/2I_1 \tag{22}$$

$$q_1 = -I_4/I_3 + \Delta p/I_3, \tag{23}$$

$$q_2 = -I_6/I_5 + \Delta p/2I_5, \tag{24}$$

where

$$I_1 = 4\bar{\mu}_2 \int_0^1 \frac{1}{h^4-(1-\bar{\mu}_2)\,h_1^4}\,dx, \tag{25}$$

$$I_2 = 4\bar{\mu}_2 \int_0^1 \frac{h^2}{h^4-(1-\bar{\mu}_2)\,h_1^4}\,dx, \tag{26}$$

$$I_3 = 4\bar{\mu}_2 \int_0^1 \frac{1}{h_1^2\{h^2-(1-\frac{1}{2}\bar{\mu}_2)\,h_1^2\}}\,dx, \tag{27}$$

$$I_4 = 4\bar{\mu}_2 \int_0^1 \frac{1}{h^2-(1-\frac{1}{2}\bar{\mu}_2)\,h_1^2}\,dx, \tag{28}$$

$$I_5 = 4\bar{\mu}_2 \int_0^1 \frac{1}{(h^2-h_1^2)^2}\,dx, \tag{29}$$

$$I_6 = 4\bar{\mu}_2 \int_0^1 \frac{1}{h^2-h_1^2}\,dx. \tag{30}$$

From equations (22), (23) and (24) we get the equation determining ϵ_1 as follows,

$$\frac{I_6}{I_5}+\frac{I_4}{I_3}-\frac{I_2}{I_1} = \Delta p\left\{\frac{1}{2I_5}+\frac{1}{I_3}-\frac{1}{2I_1}\right\}. \tag{31}$$

It can be seen by direct substitution that $h_1 = \alpha h$ satisfies this equation and hence, from equations (1) and (18),

$$\epsilon_1 = \alpha\epsilon. \tag{32}$$

Now, using the viscosity function given in equation (17), the expressions $\bar{Q}$ and F from equations (14) and (16) can be written, after noting equations (11) and (22) as

$$\bar{Q} = 1+\frac{\epsilon^2}{2}-\frac{I_2}{I_1}+\frac{\Delta p}{2I_1}, \tag{33}$$

$$F = 2I_7-2\frac{I_2^2}{I_1}+\Delta p\,\frac{I_2}{I_1}, \tag{34}$$

where I_1, I_2 are given by equations (25) and (26) and

$$I_7 = 4\bar{\mu}_2 \int_0^1 \frac{h^4}{h^4-(1-\bar{\mu}_2)\,h_1^4}\,dx. \tag{35}$$

After using equation (32) in equations (33) and (34) and evaluating the integrals given in equations (25), (26) and (35), the final expressions for $\bar{Q}$ and F can be obtained as

$$\bar{Q} = \frac{16\epsilon^2-\epsilon^4}{2(2+3\epsilon^2)}+\Delta p\,.\,g\,\frac{2(1-\epsilon^2)^{\frac{7}{2}}}{2+3\epsilon^2}, \tag{36}$$

$$F = \frac{1}{g}\left[1-\frac{2(1-\epsilon^2)^{\frac{1}{2}}}{2+3\epsilon^2}\right]+\Delta p\,\frac{2(1-\epsilon^2)^2}{2+3\epsilon^2}, \tag{37}$$

where

$$g = \frac{1-(1-\bar{\mu}_2)\,\alpha^4}{8\bar{\mu}_2} = \frac{\alpha^4}{8}+\frac{1-\alpha^4}{8\bar{\mu}_2}. \tag{38}$$

When $\bar{\mu}_2 = 1$, equation (36) reduces to the result of Shapiro *et al.* (1969).

Since the function g increases as $\bar{\mu}_2$ or α decreases ($\bar{\mu}_2, \alpha$ being less than unity), it may be noted from equations (36) and (37) that $\bar{Q}$ increases and F decreases with the

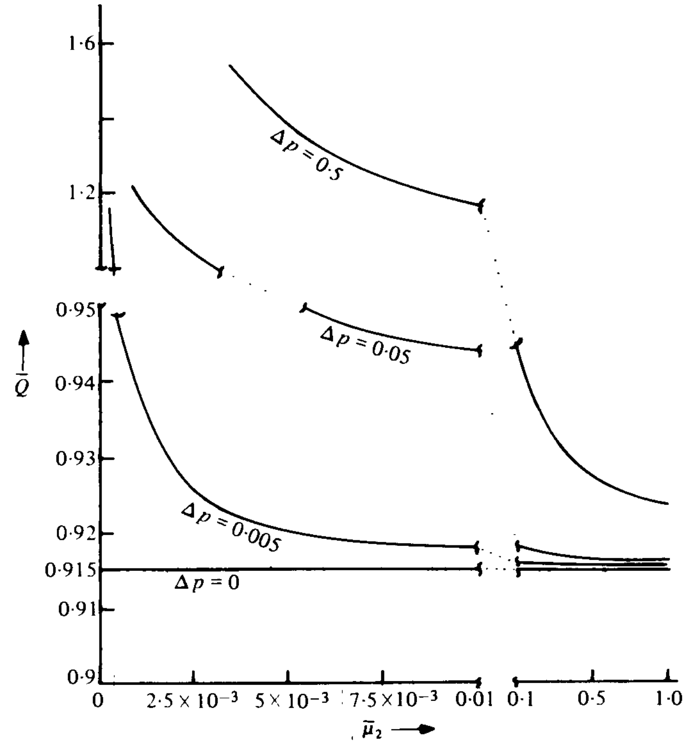

FIGURE 3. Variation of $\bar{Q}$ with $\bar{\mu}_2$ for different $\Delta\phi$. $\epsilon = 0{\cdot}6$, $\alpha = 0{\cdot}92$.

decrease in $\bar{\mu}_2$ or α, for $\Delta p > 0$. For $\Delta p = 0$, $\bar{Q}$ does not depend upon $\bar{\mu}_2$ or α, but F decreases as $\bar{\mu}$ or α decreases.

As expected, the variations of $\bar{Q}$ and F with Δp are linear for given $\bar{\mu}_2, \epsilon$. Following Shapiro *et al.* (1969) and considering the limiting case, $\bar{Q} = 0$, the pressure rise $(-\Delta p)$ can be found from equation (36) as,

$$(-\Delta p)|_{Q=0} = \frac{1}{g}\frac{16\epsilon^2 - \epsilon^4}{4(1-\epsilon^2)^{\frac{1}{2}}}, \tag{39}$$

which decreases as $\bar{\mu}_2$ decreases and varies as ϵ^2/g for $\epsilon \ll 1$. Again, when $F = 0$, it may be noted from equation (37) that the pressure rise $(-\Delta p)|_{F=0}$ has the same behaviour as mentioned above.

To see the effects of ϵ on the flow characteristics analytically, equations (36) and (37) can be approximated for $\epsilon \ll 1$ as

$$\left.\begin{aligned} \bar{Q} &= \epsilon^2(4 - 5\Delta p \,.\, g) + \Delta p, \\ F &= \epsilon^2\left(\frac{2}{g} - \tfrac{7}{2}\Delta p\right) + \Delta p. \end{aligned}\right\} \tag{40}$$

It can be seen from these equations that $\bar{Q}$ and F increase as ϵ increases for $\Delta p = 0$ or for $4/7g > \Delta p$ as $g \geqslant \frac{1}{8}$.

To see these results quantitatively, the expressions of $\bar{Q}$ and F from equations (36) and (37) have been plotted in figures 3, 4 and 5 for various values of $\bar{\mu}_2$, ϵ and Δp. From these graphs, the results discussed above may again be noted. In particular, it is seen from figures 3 and 5 that $\bar{Q}$ and F increase with $\Delta p > 0$ but the difference in $\bar{Q}$ from its corresponding value for $\Delta p = 0$ is large for $\bar{\mu}_2 \ll 1$ than that of $\bar{\mu}_2 = 1$ even

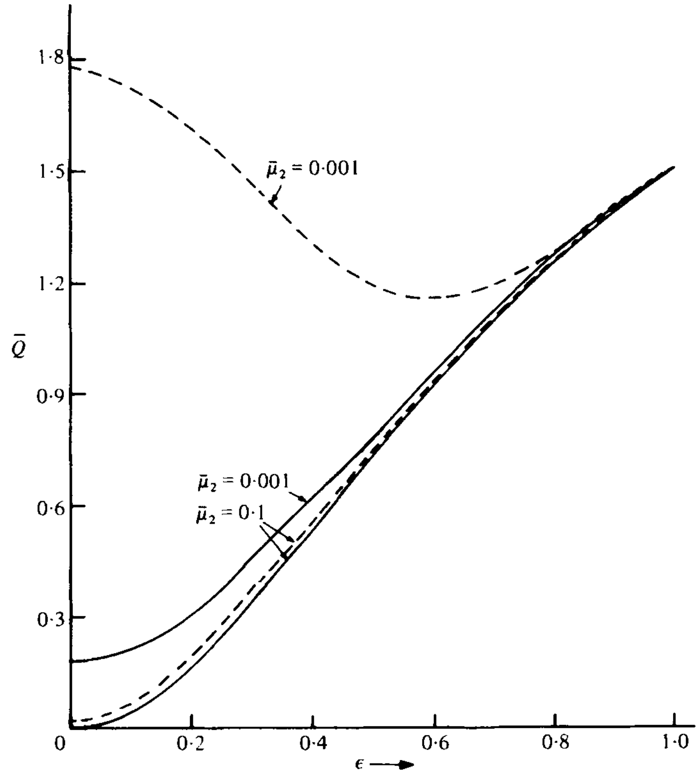

FIGURE 4. Variation of $\bar{Q}$ with ϵ for different Δp and $\bar{\mu}_2$. $\alpha = 0·92$. – – –, $\Delta p = 0·05$; ——, $\Delta p = 0·005$.

for very small Δp ($= 0·005$, say) whereas the difference in F remains constant for all $\bar{\mu}_2$. From figures 4 and 5, it is noted that $\bar{Q}$, F may increase with ϵ only under certain condition which depends upon Δp and $\bar{\mu}_2$.

4. Application

Let us apply the circular-tube model to the chyme movement through small intestines, by considering the chyme core to be surrounded by a thin layer of mucus (peripheral layer) (Guyton 1971). It may be noted that the viscosity of chyme which may be considered as a semi-fluid, is much greater than the mucus viscosity. During peristalsis, the chyme core takes the same shape as that of intestine and is assumed to be given by equation (18). As reported by Barton & Raynor (1968) the observed average chyme velocity ($Q'/\pi a^2 = \bar{Q}c$) is of the order 2·54 cm min^{-1} while the calculated chyme velocity is 1·83 cm min^{-1}.

To compare our analysis with their observations, we use the values of the parameters for the chyme transport as given by Barton & Raynor (1968), which are as follows:

$$a = 1·25 \text{ cm}; \quad c = 2 \text{ cm min}^{-1}; \quad \lambda = 8·01 \text{ cm}; \quad a/\lambda = 0·156.$$

The mucus-layer thickness is of the order 0·1 cm and b may be taken as 0·75 cm, giving $\epsilon = 0·6$, $\alpha = 0·92$ (Guyton 1971). The viscosity of the gastric mucus varies as $1 \sim 10^2$ cP (Janowitz & Hollander 1954; Heatley 1959; Snary, Allen & Pain 1971).

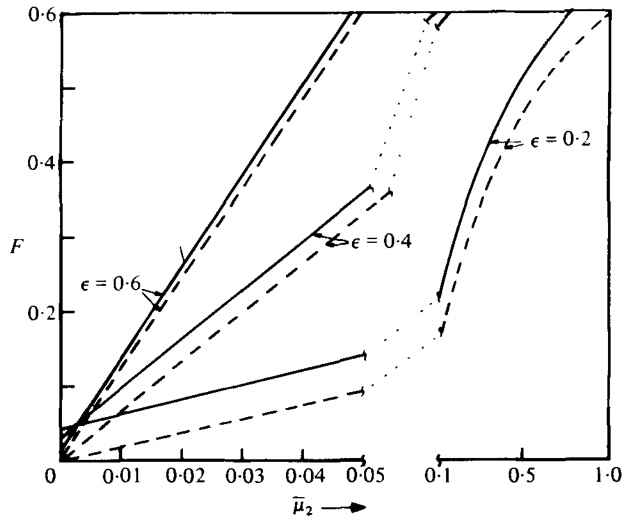

FIGURE 5. Variation of F with $\bar{\mu}_2$ for different Δp and ϵ; $\alpha = 0{\cdot}92$. ---, $\Delta p = 0$; ——, $\Delta p = 0{\cdot}05$.

The viscosity of the chyme may be of the same order of magnitude as that of faeces which varies as $10^3 \sim 10^6$ cP (Patel, Picologlou & Lykoudis 1973; Picologlou, Patel & Lykoudis 1973). Considering these values, $\bar{\mu}_2$ may be taken as to vary 10^{-4}–10^{-2}.

Using the above data, $\bar{Q}$ can be calculated from equation (36) as 1·157 for $\bar{\mu}_2 = 10^{-4}$, $\Delta p = 0{\cdot}005$. The average chyme velocity is then calculated as 2·314 cm/min which differs from the observed value by only 9% whereas for $\Delta p = 0{\cdot}005$, $\bar{\mu}_2 = 1$, this difference is nearly 27% (Barton & Raynor 1968).

For $\Delta p = 0$, $\epsilon = 0{\cdot}6$, $\alpha = 0{\cdot}92$, the friction force, F, can be calculated from equation (37) as 3·844 for $\bar{\mu}_2 = 1$ and $1{\cdot}45 \times 10^{-3}$ for $\bar{\mu}_2 = 10^{-4}$ which is negligible in comparison with the value for $\bar{\mu}_2 = 1$, showing the importance of peripheral-layer viscosity on friction force (see figure 5).

5. Peristaltic motion in a channel

Consider the symmetrical peristaltic flow of a fluid in a channel as shown in figure 6. In this case also, the viscosity function in the dimensionless form has been assumed to be given by equation (17), r being replaced by z.

Following, the same procedure as in the previous case the dimensionless flux per unit width, $\bar{Q}$ ($= Q'/2ac$, Q' is the flux in the stationary co-ordinate system) and friction force per unit width, F ($= F'a/\mu_1\lambda c$; F' is the friction force at the upper wall in the stationary co-ordinate system) can be obtained as

$$\bar{Q} = \frac{3\epsilon^2}{2+\epsilon^2} + \Delta p g_1 \frac{2(1-\epsilon^2)^{\frac{5}{2}}}{2+\epsilon^2}, \tag{41}$$

$$F = \frac{1}{g_1}\frac{\epsilon^2}{(2+\epsilon^2)(1-\epsilon^2)^{\frac{1}{2}}} + \Delta p \frac{2(1-\epsilon^2)}{2+\epsilon^2}, \tag{42}$$

where
$$g_1 = \frac{1-(1-\bar{\mu}_2)\alpha^3}{3\bar{\mu}_2} = \frac{\alpha^3}{3} + \frac{1-\alpha^3}{3\bar{\mu}_2}. \tag{43}$$

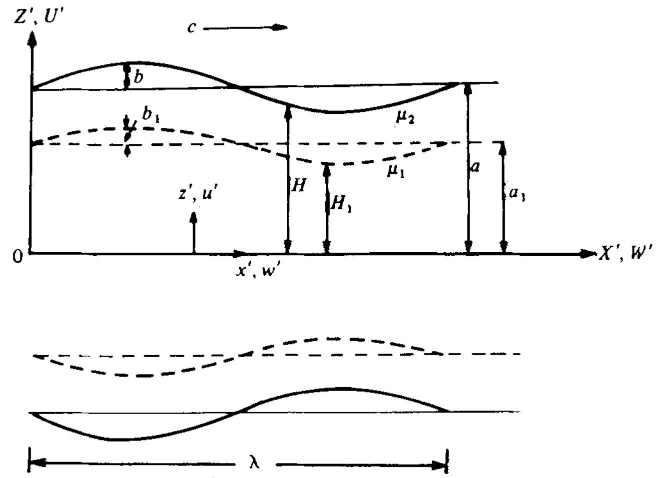

FIGURE 6. Peristaltic motion in a channel with a peripheral layer adjacent to the wall.

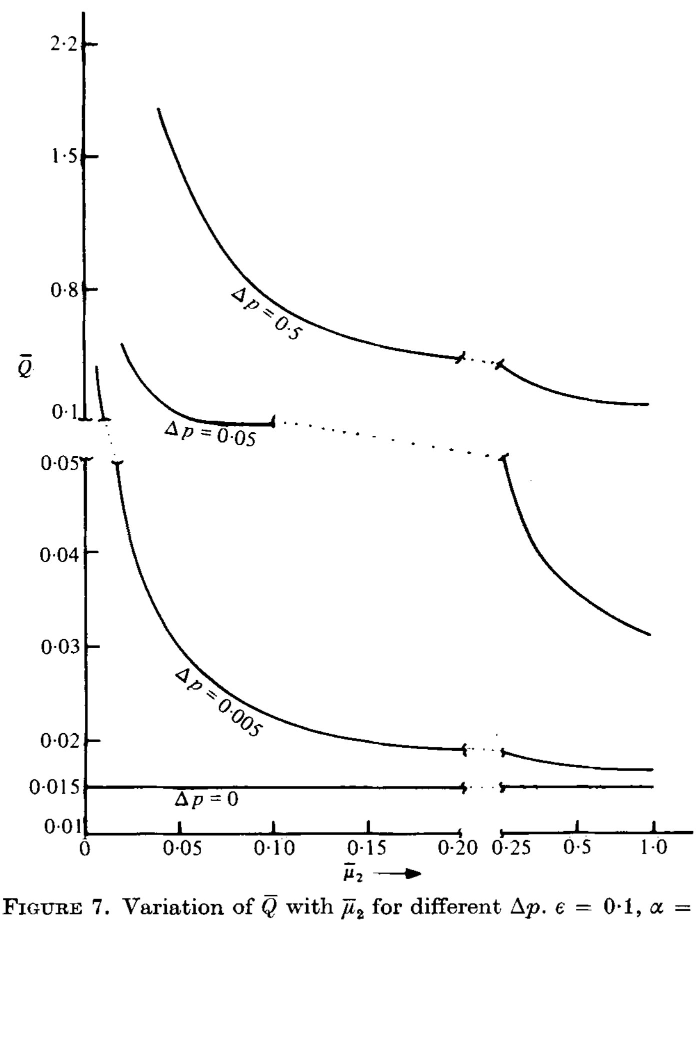

FIGURE 7. Variation of $\bar{Q}$ with $\bar{\mu}_2$ for different Δp. $\epsilon = 0{\cdot}1$, $\alpha = 0{\cdot}833$.

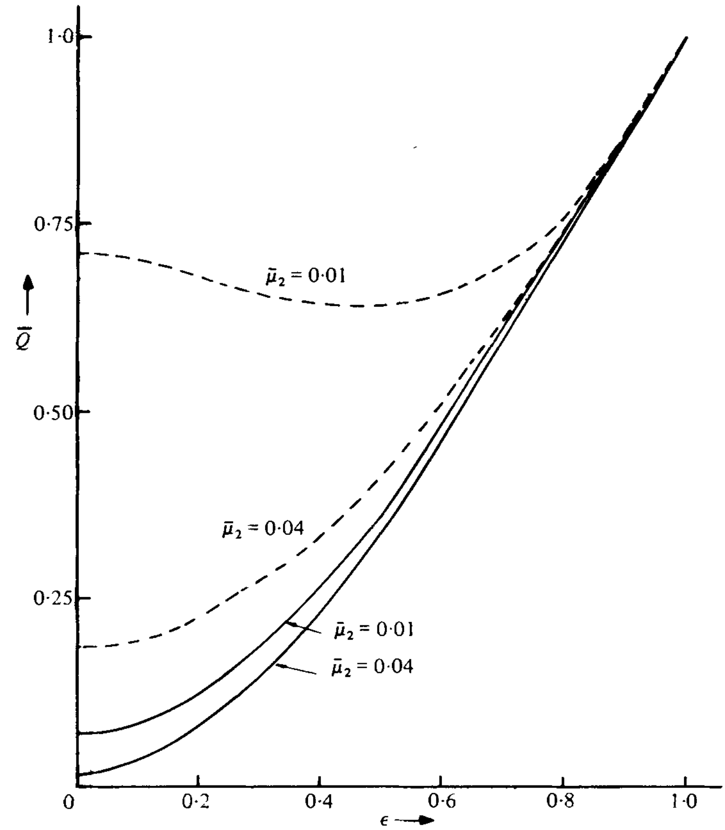

FIGURE 8. Variation of $\bar{Q}$ with ϵ for different Δp and $\bar{\mu}_2$. $\alpha = 0{\cdot}833$. – – –, $\Delta p = 0{\cdot}05$; ——, $\Delta p = 0{\cdot}005$.

For $\bar{\mu}_2 = 1$, it may be noted that equation (41) reduces to the same form as equation (30) of Lardner & Shack (1972) when the eccentricity of the elliptical motion of cilia tips is zero in their analysis (Shapiro *et al.* 1969).

Since the behaviour of the function, g_1 with $\bar{\mu}_2$ and α is same as the function, g of equation (38), all the results discussed in the circular case regarding $\bar{Q}$ and F are qualitatively valid in this case also. To see these results quantitatively, the expressions of $\bar{Q}$ and F from equations (41) and (42) have been plotted in figures 7, 8 and 9 for different values of $\bar{\mu}_2$, Δp and ϵ. From figures 7 and 9 it is noted that $\bar{Q}$ increases and F decreases as $\bar{\mu}_2$ decreases for $\Delta p > 0$. Further, it can also be observed that $\bar{Q}$ and F increase with the increase in ϵ for certain values of $\bar{\mu}_2$ and Δp (see figures 8 and 9). For $\Delta p = 0$, $\bar{Q}$ does not depend upon $\bar{\mu}_2$ but F decreases as $\bar{\mu}_2$ decreases and its value for $\bar{\mu}_2 \ll 1$ is much smaller than its corresponding value for $\bar{\mu}_2 = 1$ (see figure 9).

The above analysis may also be applied in the study of flow rates observed in the *ductus efferentes* of the male reproductive tract. As pointed out by Lardner & Shack (1972), the approximately estimated value of flow rate of human rete testis fluid per *ductus efferent* is of the order 6×10^{-3} ml/hr.

The viscosity of the bull semen is of the order of 25 cP (Glover & Scott Blair 1966) and hence the viscosity of the bio-fluid in the core region may be taken as of the same order. Considering the peripheral-layer viscosity as 1 cP (i.e. of water), $\bar{\mu}_2$ can be

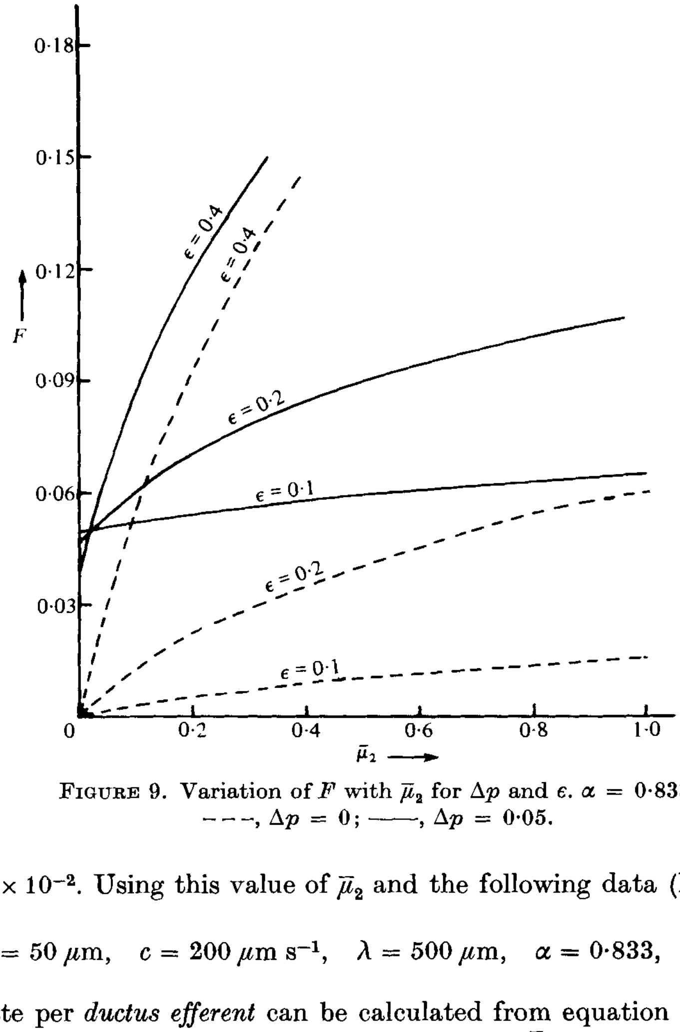

FIGURE 9. Variation of F with $\bar{\mu}_2$ for Δp and ϵ. $\alpha = 0{\cdot}833$.
- - -, $\Delta p = 0$; ——, $\Delta p = 0{\cdot}05$.

found as 4×10^{-2}. Using this value of $\bar{\mu}_2$ and the following data (Lardner & Shack 1972),

$$a = 50\,\mu\text{m}, \quad c = 200\,\mu\text{m s}^{-1}, \quad \lambda = 500\,\mu\text{m}, \quad \alpha = 0{\cdot}833, \quad \epsilon = 0{\cdot}1,$$

the flow rate per *ductus efferent* can be calculated from equation (41) as $\bar{Q} = 0{\cdot}199$ for $\Delta p = 0{\cdot}05$ which gives the flow rate as Q' $(= \bar{Q} \times \pi a^2 c) = 1{\cdot}09 \times 10^{-3}$ ml h^{-1}. This value is in error from the estimated value by 82%. However, calculating the flux from equation (30) of Lardner & Shack (1972) for the same pressure drop, it may be noted this error is 96·5%.

6. Conclusion

The effects of peripheral-layer viscosity on the transport of a bio-fluid due to peristaltic motion have been discussed by considering circular-tube and parallel-plates models under the long wavelength approximation. It has been shown that the flow flux increases and friction force decreases as the thickness of the peripheral layer increases or its fluid viscosity decreases. Even for a small value of pressure drop, the increase in the flow or decrease in the friction force is considerable for smaller peripheral-layer viscosity in comparison to their respective values when the peripheral-layer viscosity is equal to the central-layer viscosity.

For zero pressure drop, it has been noted that the flux does not depend upon the peripheral-layer viscosity but the force of friction decreases considerably with its decrease.

The analysis has been applied and compared with the observed flow rates in intestine and *ductus efferentes* of the male reproductive tract, and the importance of the peripheral-layer viscosity has been pointed out.

The authors are extremely thankful to Professor Sir James Lighthill for his valuable comments and suggestions regarding this paper.

REFERENCES

Barton, C. & Raynor, S. 1968 *Bull. Math. Biophys.* **30**, 663–680.

Boyarsky, S. & Labay, P. 1972 *Ureteral Dynamics*. Baltimore: Williams and Wilins.

Burns, J. C. & Parkes, T. 1967 *J. Fluid Mech.* **29**, 731–734.

Chow, T. S. 1970 *J. Appl. Mech.* **37**, 901–905.

Fung, Y. C. & Yih, C. S. 1968 *J. Appl. Mech.* **35**, 669–675.

Glover, F. A. & Scott Blair, G. W. 1966 *Biorheology* **3**, 189–194.

Gupta, B. B. & Seshadri, V. 1976 *J. Biomech.* **9**, 105–109.

Guyton, A. C. 1971 *Medical Physiology*. Philadelphia: Saunders.

Hanin, M. 1968 *Israel J. Technol.* **6**, 67–71.

Heatley, N. G. 1959 *Gastroenterology* **37**, 304–308.

Jaffrin, M. Y. & Shapiro, A. H. 1971 Peristaltic pumping. *Ann. Rev. Fluid Mech.* **3**, 13–36.

Janowitz, H. D. & Hollander, F. 1954 *Gastroenterology* **26**, 582–585.

Lardner, T. J. & Shack, W. J. 1972 *Bull. Math. Biophys.* **34**, 325–335.

Latham, T. W. 1966 Fluid motions in the peristaltic pump. M.S. thesis, Massachusetts Institute of Technology.

Li, C. H. 1970 *J. Biomech.* **3**, 513–523.

Lykoudis, P. S. & Roos, R. 1970 *J. Fluid Mech.* **43**, 661–674.

Mittra, T. K. 1971 Effect of wall properties and poiseuille flow on peristalsis. Ph.D. dissertation, University of Mississippi.

Mittra, T. K. & Prasad, S. N. 1974 *Bull. Math. Biology* **36**, 127–141.

Patel, P. D., Picologlou, B. F. & Lykoudis, P. S. 1973 *Biorheology* **10**, 441–445.

Picologlou, B. F., Patel, P. D. & Lykoudis, P. S. 1973 *Biorheology* **10**, 431–440.

Shapiro, A. H., Jaffrin, M. Y. & Weinberg, S. L. 1969 *J. Fluid Mech.* **37**, 799–825.

Sleigh, M. A. 1974 *Cilia and Flagella*. Academic.

Smelser, R. E., Shack, W. J. & Lardner, T. J. 1974 *J. Biomech.* **7**, 349–355.

Snary, D., Allen, A. & Pain, R. H. 1971 *Europ. J. Biochem.* **24**, 183–189.

Tong, P. & Vawter, D. 1972 *J. Appl. Mech.* **39**, 857–862.

Weinberg, S. L., Eckstein, E. C. & Shapiro, A. H. 1971 *J. Fluid Mech.* **49**, 461–479.

Yin, F. & Fung, Y. C. 1969 *J. Appl. Mech.* **36**, 579–587.

Zien, T. F. & Ostrach, S. A. 1970 *J. Biomech.* **3**, 63–75.